Laplace transforms is something that Mathematica is good at. Shifting, as far as I can tell, is something that can be routinely shined on.

1 - 16 Laplace transforms
Find the transform. Assume that a, b, ω, θ are constants.

1. $3 t + 12$

```
ClearAll["Global`*"]
e1 = LaplaceTransform[3 t + 12, t, s]
```

$$\frac{3}{s^2} + \frac{12}{s}$$

The correct answer.

3. $\text{Cos}[\pi t]$

```
ClearAll["Global`*"]
e1 = LaplaceTransform[Cos[π t], t, s]
```

$$\frac{s}{\pi^2 + s^2}$$

The correct answer.

5. $e^{2t}\,\text{Sinh}[t]$

```
ClearAll["Global`*"]
e1 = LaplaceTransform[ⅇ^(2 t) Sinh[t], t, s]
```

$$\frac{1}{3 - 4 s + s^2}$$

The correct answer.

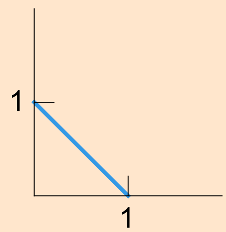
7. $\text{Sin}[\omega t + \theta]$

```
ClearAll["Global`*"]
e1 = LaplaceTransform[Sin[ω t + θ], t, s]
```

$$\frac{\omega \,\text{Cos}[\theta] + s\,\text{Sin}[\theta]}{s^2 + \omega^2}$$

The correct answer.

9.




```
ClearAll["Global`*"]
```

```
e1 = f[t_] = -t + 1
```

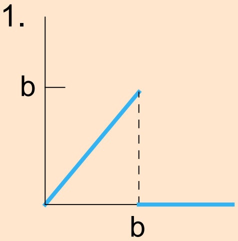
```
1 - t
```

```
e4 = LaplaceTransform[If[t > 0 && t < 1, 1 - t, 0], t, s]
```

$$\frac{-1 + e^{-s} + s}{s^2}$$

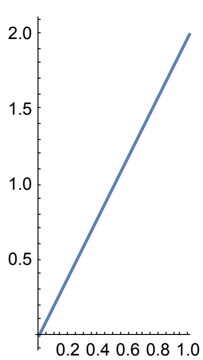
This is the right answer. There is a big difference on what comes out of the Laplace Transform based on whether the domain is restricted or not.

11.




```
ClearAll["Global`*"]
```

```
Plot[ 2 x, {x, 0, 1}, PlotRange → Automatic,
  ImageSize → 100, AspectRatio → Automatic]
```



```
e1 = t
```

t

```
e2 = Simplify[LaplaceTransform[If[t > 0 && t < b, t, 0], t, s]]
```

$$\begin{cases} \frac{e^{-b s}\left(-1+e^{b s}-b s\right)}{s^2} & b > 0 \\ 0 & \text{True} \end{cases}$$

$$e3 = \frac{e^{-b s}\left(-1+e^{b s}-b s\right)}{s^2}$$

$$\frac{e^{-b s}\left(-1+e^{b s}-b s\right)}{s^2}$$

$$e4 = e3 \;/.\; \left(e^{-b s}\left(-1+e^{b s}-b s\right)\right) \to \text{Expand}\left[e^{-b s}\left(-1+e^{b s}-b s\right)\right]$$

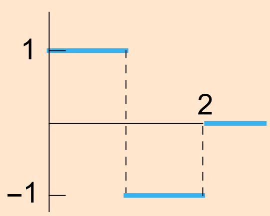
$$\frac{1-e^{-b s}-b\, e^{-b s}\, s}{s^2}$$

$$e5 = e4 \;/.\; \frac{1-e^{-b s}-b\, e^{-b s}\, s}{s^2} \;\text{->}\; \frac{1-e^{-b s}}{s^2} - \frac{b\, e^{-b s}\, s}{s^2}$$

$$\frac{1-e^{-b s}}{s^2} - \frac{b\, e^{-b s}}{s}$$

Above: This is the text answer.

13.




```
ClearAll["Global`*"]
```

```
e1 = LaplaceTransform[
  Piecewise[{{1, t > 0 && t < 1}, {-1, t > 1 && t < 2}}], t, s]
```

$$-\frac{e^{-2 s}\left(-1+e^{s}-e^{2 s}+e^{2 s}\,\text{Cosh}[s]-e^{2 s}\,\text{Sinh}[s]\right)}{s}$$

```
e2 = Simplify[e1]
```

$$\frac{e^{-2 s}\left(-1+e^{s}\right)^2}{s}$$

$$\texttt{PossibleZeroQ}\left[\frac{e^{-2 s} (-1+e^{s})^{2}}{s}-\frac{(1-e^{-s})^{2}}{s}\right]$$

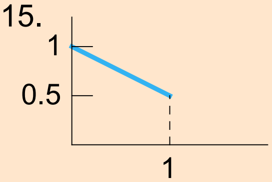
True

The above shows that Mathematica's answer and the text answer are equivalent. With a ‘sleight’ maneuver I could even do

$$\texttt{e6} = \texttt{e2} \;/.\; e^{-2 s} (-1+e^{s})^{2} \;\texttt{->}\; (1-e^{-s})^{2}$$

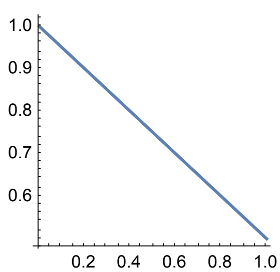
$$\frac{(1-e^{-s})^{2}}{s}$$

15.




```
ClearAll["Global`*"]
```

$$\texttt{Plot}\left[-\frac{1}{2} x + 1, \{x, 0, 1\}, \texttt{PlotRange} \to \texttt{Automatic}, \texttt{ImageSize} \to 140, \texttt{AspectRatio} \to 1\right]$$



$$\texttt{e1} = \texttt{LaplaceTransform}\left[\texttt{Piecewise}\left[\left\{\left\{-\frac{1}{2} t + 1, t > 0 \;\&\&\; t < 1\right\}, \{0, t > 1 \;\&\&\; t < 0\}\right\}\right], t, s\right]$$

$$\frac{e^{-s} (1 - e^{s} - s + 2 e^{s} s)}{2 s^{2}}$$

$$\texttt{e2} = \texttt{TrigToExp}[\texttt{e1}]$$

$$-\frac{1}{2 s^{2}} + \frac{e^{-s}}{2 s^{2}} + \frac{1}{s} - \frac{e^{-s}}{2 s}$$

The above answer is correct.

21. Nonexistence. Give sample examples of functions (defined for all $t \geq 0$) that have no Laplace transform.

If the following expression is true for some constants M and k it satisfies the "growth restriction"

$$| f(t) \le M e^{k t} |$$

The sol'n text gives e^{t^2} as an example. The expression above is a more general guide. The *k* and *M* are just constants, so the *lhs* can overwhelm them if it has an exponential or similar nature.

25 - 32 Inverse Laplace transforms
Given F(s) = L(f), find f(t). Here a, b, L, n, are constants.

25. $\frac{0.2 s + 1.8}{s^2 + 3.24}$

```
ClearAll["Global`*"]
```

$$e1 = \text{InverseLaplaceTransform}\left[\frac{0.2 s + 1.8}{s^2 + 3.24}, s, t\right]$$

$$(0.5 + 0.1 i) e^{(0. -1.8 i) t} \left((0.384615 + 0.923077 i) - (0. + 1. i) e^{(0. +3.6 i) t}\right)$$

```
e2 = FullSimplify[e1]
```

$$e^{(0. -1.8 i) t} \left((0.1 + 0.5 i) + (0.1 - 0.5 i) e^{(0. +3.6 i) t}\right)$$

```
e3 = ExpToTrig[e2]
```

(Cos[(1.8 + 0. i) t] - i Sin[(1.8 + 0. i) t])
 ((0.1 + 0.5 i) + (0.1 - 0.5 i) (Cos[(3.6 + 0. i) t] + i Sin[(3.6 + 0. i) t]))

```
e4 = FullSimplify[e3]
```

0.2 Cos[1.8 t] + 1. Sin[1.8 t]

The above answer matches the text. A bit of work to recast it.

27. $\frac{s}{L^2 s^2 + n^2 \pi^2}$

```
ClearAll["Global`*"]
```

$$e1 = \text{InverseLaplaceTransform}\left[\frac{s}{L^2 s^2 + n^2 \pi^2}, s, t\right]$$

$$\frac{\text{Cos}\left[\frac{n \pi t}{L}\right]}{L^2}$$

The above answer matches the text.

29. $\frac{12}{s^4} - \frac{228}{s^6}$

```
ClearAll["Global`*"]
```

```
e1 = InverseLaplaceTransform[12/s^4 - 228/s^6, s, t]
```

$2\,t^3 - \frac{19\,t^5}{10}$

The above answer matches the text. Here is demonstrated the linear nature of L^{-1}: separate fractions can be calculated as a single operand.

31. $\frac{s + 10}{s^2 - s - 2}$

```
ClearAll["Global`*"]
```

```
e1 = InverseLaplaceTransform[(s + 10)/(s^2 - s - 2), s, t]
```

$-3\,e^{-t} + 4\,e^{2\,t}$

The above answer matches the text.

33 - 45 Application of s-shifting
In problems 33 - 36 find the transform. In problems 37 - 45 find the inverse transform.

33. $t^2\,e^{-3\,t}$

```
ClearAll["Global`*"]
```

```
e1 = LaplaceTransform[t^2 e^(-3 t), t, s]
```

$\frac{2}{(3 + s)^3}$

The above answer matches the text.

35. $0.5\,e^{-4.5\,t}\,\mathrm{Sin}[2\,\pi\,t]$

```
ClearAll["Global`*"]
```

```
e1 = LaplaceTransform[0.5 e^(-4.5 t) Sin[2 π t], t, s]
```

$\frac{3.14159}{4\,\pi^2 + (4.5 + s)^2}$

The above answer matches the text.

37. $\frac{\pi}{(s+\pi)^2}$

```
ClearAll["Global`*"]
```

e1 = InverseLaplaceTransform$\left[\frac{\pi}{(s+\pi)^2}, s, t\right]$

$e^{-\pi t}\,\pi\,t$

The above answer matches the text.

39. $\frac{21}{\left(s+\sqrt{2}\right)^4}$

```
ClearAll["Global`*"]
```

e1 = InverseLaplaceTransform$\left[\frac{21}{\left(s+\sqrt{2}\right)^4}, s, t\right]$

$\frac{7}{2}\,e^{-\sqrt{2}\,t}\,t^3$

The above answer matches the text.

41. $\frac{\pi}{s^2 + 10\,\pi\,s + 24\,\pi^2}$

```
ClearAll["Global`*"]
```

e1 = InverseLaplaceTransform$\left[\frac{\pi}{s^2 + 10\,\pi\,s + 24\,\pi^2}, s, t\right]$

$\frac{1}{2}\,e^{-6\,\pi\,t}\left(-1 + e^{2\,\pi\,t}\right)$

e2 = FullSimplify$\left[\text{ExpToTrig}\left[\frac{\left(-1 + e^{2\,\pi\,t}\right)}{2\;e^{\pi\,t}}\right]\right]$

Sinh[π t]

e3 = $e^{-5\,\pi\,t}$ e2

$e^{-5\,\pi\,t}$ Sinh[π t]

The above answer matches the text.

43. $\frac{2s-1}{s^2-6s+18}$

```
ClearAll["Global`*"]
```

e1 = InverseLaplaceTransform$\left[\frac{2s-1}{s^2-6s+18}, s, t\right]$

$\frac{1}{6} e^{(3-3i)t}\left((6+5i)+(6-5i)e^{6it}\right)$

```
e2 = FullSimplify[e1]
```

$\frac{1}{3} e^{3t}(6\,\mathrm{Cos}[3t]+5\,\mathrm{Sin}[3t])$

The above answer matches the text.

45. $\frac{k_0(s+a)+k_1}{(s+a)^2}$

```
ClearAll["Global`*"]
```

e1 = InverseLaplaceTransform$\left[\frac{k_0(s+a)+k_1}{(s+a)^2}, s, t\right]$

$e^{-at}(k_0+t\,k_1)$

The above answer matches the text.

The problems since No. 33 demonstrate that S-shifting doesn’t really exist for Mathematica user. Just put the expression in and turn the crank.